LOYOLA COLLEGE (AUTONOMOUS) CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION – **MATHEMATICS**

FIRST SEMESTER – **APRIL 2025**

PMT1MC06 – PROBABILITY THEORY AND RANDOM PROCESSES

Date: 03-05-2025 Dept. No. Max. : 100 Marks

Time: 09:00 AM - 12:00 PM

SECTION A – K1 (CO1)

	Answer ALL the questions (5 x 1 = 5)
1	**Answer the following**
a)	If a pair of dice is tossed and X denotes the sum of the numbers on them, then discover the expectation of X.
b)	Write the formula for the partial correlation coefficient between x_2 and x_3 with respect to x_1.
c)	State invariance property of consistent estimator.
d)	Define statistical hypothesis.
e)	Recall the concept of autocorrelation.

SECTION A – K2 (CO1)

	Answer ALL the questions (5 x 1 = 5)
2	**MCQ**
a)	For two random variables X and Y, $var(aX + bY)$ is equal to (i) $a^2\sigma_X^2 + b^2\sigma_Y^2 + 2ab\,Cov(X,Y)$ (ii) $a^2\sigma_X^2 + b^2\sigma_Y^2 + 2ab\,\gamma\,\sigma_x\sigma_y$ (iii) Both (i) and (ii) (iv) None
b)	If the regression lines are perpendicular to each other, then the value of γ is (i) -1 (ii) +1 (iii) 0 (iv) None
c)	Which two of the following are correct? A sufficient estimator is necessarily: (i) unbiased (ii) consistent (iii) most efficient (iv) None
d)	Identify the symbol used for the level of significance: (i) α (ii) β (iii) $1-\alpha$ (iv) $1-\beta$
e)	Which of the following is not true? (i) A second order stationary process is also first order stationary (ii) For a second order stationary process, the autocorrelation function is a function of time difference (iii) The wide sense stationary process is also known as strongly stationary process (iv) A first order stationary random process has a constant mean.

SECTION B – K3 (CO2)

	Answer any THREE of the following (3 x 10 = 30)
3	State and prove Khintchin's theorem.
4	In 10 areas, the infant-mortality (y) and birth rates (x) are obtained as follows. Calculate the correlation coefficient.

X	22.9	17.8	20.8	21.3	20.7	20.9	17.5	13.6	23.3	18.1
Y	44	46	56	42	32	47	38	45	41	52

5	State and prove the sufficient conditions for consistent estimators.
6	In a shipment of 10 articles, θ are defective. The hypothesis $H_0: \theta = 5$ is rejected in favour $H_1: \theta = 4$ if (i) two articles selected at random with replacement are both of the same type either defective or

	non-defective; or (ii) if two particles selected at random with replacement are having the combination of one defective and one non-defective. Determine the size of the critical regions and power of the test in both the cases.
7	Show that the random process $X(t) = A\cos(\omega t + \theta)$ is wide-sense stationary, where A and ω are constants and θ is uniformly distributed on the interval $(0,2\pi)$.
	SECTION C – K4 (CO3)
	Answer any TWO of the following **(2 x 12.5 = 25)**
8	State and prove weak law of large numbers. In addition, discuss its application through an example.
9	Show that the coefficient of correlation γ between two variables x and y is $\gamma = \frac{(\sigma_x^2+\sigma_y^2-\sigma_{x-y}^2)}{2\sigma_x\sigma_y}$. Apply this result in calculating the coefficient of correlation between heights (in inches) of brothers and sisters for the following data:
10	For a random sampling from a normal population, estimate the maximum likelihood estimators for: (i) The population mean, when population variance is known. (ii) The population variance, when population mean is known. (iii) The simultaneous estimation of both the population mean and the population variance.
11	Write short notes on the following in random process: (i) State space (ii) Markov process (iii) stationary process (iv) Deterministic and (v) Non-deterministic.
	SECTION D – K5 (CO4)
	Answer any ONE of the following **(1 x 15 = 15)**
12	Determine the most powerful critical region by demonstrating the Neyman-Pearson fundamental lemma.
13	Explain the four classes of random processes through relevant real-world examples.
	SECTION E – K6 (CO5)
	Answer any ONE of the following **(1 x 20 = 20)**
14	Construct a problem related to a music event, and then apply the rank correlation method to study the similarity in music taste among three judges evaluating 12 participants.
15	Construct three estimators for a random sample (X_1, X_2, X_3) of size 3 which is drawn from a normal population with mean μ and variance σ^2. Also, verify they are unbiased or not? Further, determine the best estimator.

Data for question 9:

Family No	1	2	3	4	5	6	7	8	9	10	11
Brother (x)	71	68	66	67	70	71	70	73	72	65	66
Sister (y)	69	64	65	63	65	62	65	64	66	69	62

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